Noise Engineering **Class 2017_Fall**
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HomeWork #3 (Due 11/27)

1. Obtain a solution for the following 1-D wave equation using numerical scheme (see appendix) and compare to the analytic solution.

Equation : $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$

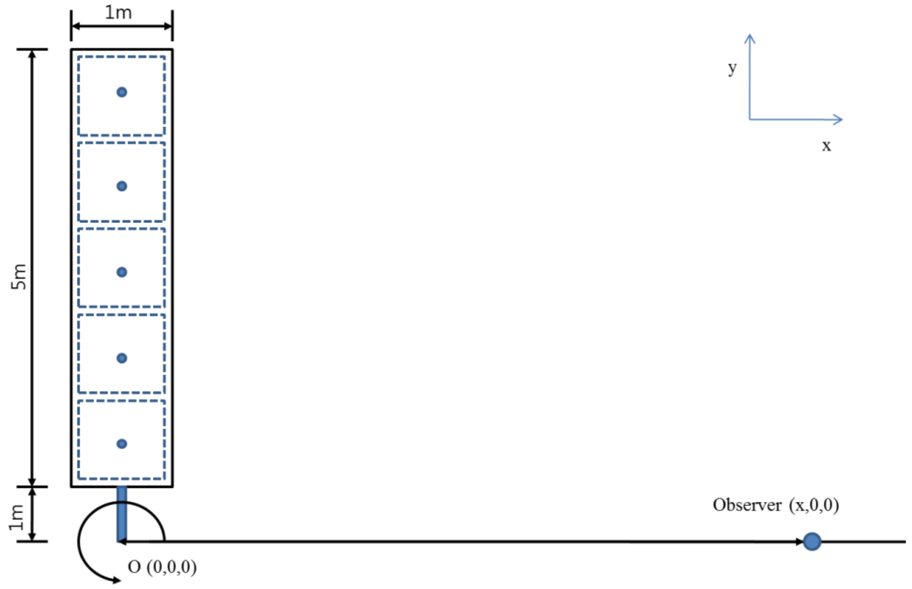
Initial condition : $u(x) = 0.3\exp\left(-ln5 \times \left(\frac{x}{5}\right)^2\right)$

2. Obtain a solution for the following 3-D wave equation using numerical scheme (see appendix) and compare to the analytic solution.

Equation : $\frac{\partial u}{\partial t} + \frac{u}{r} + \frac{\partial u}{\partial r} = 0, \quad r > 10$

Boundary condition : $r = 10, \quad u = \sin(\omega t), \quad t > 0, \quad \omega = 0.5\pi$

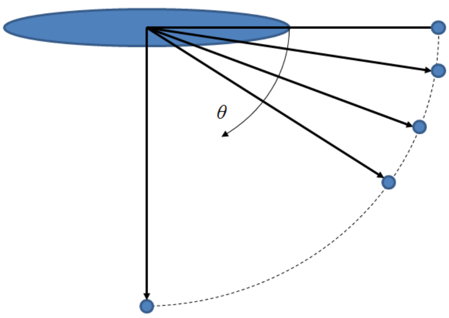
3. In order to analyze rotating machinery noise, system could be designed as follows. Assume that there are **3 blades** with an angular gap of 120 degree and **5 noise sources** P_i exist on each blade surface. Angular speed of rotor is **600RPM**, pitch angle is 7 **degree**. Surface pressure $\mathbf{p_1}$ **is equal to** $\boldsymbol{\rho U^2}$



a) Solve for the time history of the **pressure** at the observer location. Observer is located in the same surface as rotor, and x is (20+the last digit of your student ID e.g. 2009-11449 => x=29).

b) Obtain the time history of the pressure at the observer when the observer moves from (20,0,0) to (50,0,0) in the speed of **M=0.3**.

c) Assume that observer locates on the vertical (to the rotor) surface as in the figure. Obtain SPL by every 10 degree of θ and draw directivity pattern of rotating machinery noise.



Appendix

※ Numerical analysis condition

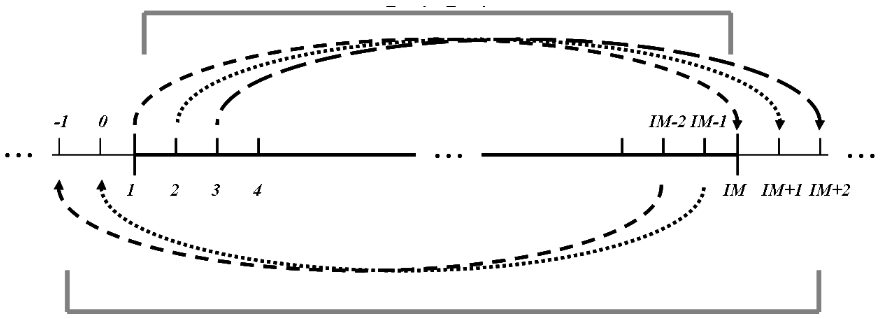
Calculation area : -200 < r < 200 (problem 3) / 10 < r < 200 (problem 4)

Time to show the results : $t = 20s, 40s, 80s$

Grid size : $\Delta x = 0.5$

Boundary condition :

Problem 3 : periodic boundary condition



U(1) = U(IM)

U(2) = U(IM+1)

U(3) = U(IM+2)

U(4) = U(IM+3)

U(0) = U(IM-1)

U(-1) = U(IM-2)

U(-2) = U(IM-3)

Problem 4:

Inlet B.C (U(0), U(-1), U(-2)) : Use the exact solution at the location and time

Outlet B.C (U(IM+1), U(IM+2), U(IM+3)) : Use linear interpolation

U(IM+1) = 2U(IM) – U(IM-1)

U(IM+2) = 2U(IM+1) – U(IM)

U(IM+3) = 2U(IM+2) – U(IM+1)

※ Numerical scheme

Space differentiation : Dispersion-Relation-Preserving

$$\left(\frac{\partial f}{\partial x}\right)_l = \frac{1}{\Delta x}\sum_{j=-3}^{3} a_j f(x + j\Delta x); \qquad a_{-j} = -a_j$$

$a_0 = 0.0$

$a_1 = -a_{-1} = 0.770882380518$

$a_2 = -a_{-2} = -0.166705904415$

$a_3 = -a_{-3} = 0.0208431427703$

Time differentiation : 4th order Optimized Adams-Bashforth method

$k_1 = f(x_n, t - 3\Delta t)$

$k_2 = f(x_n, t - 2\Delta t)$

$k_3 = f(x_n, t - \Delta t)$

$k_4 = f(x_n, t) \qquad if\ t < 0 \quad then\ f(t) = 0$

$y^{n+1} = y^n + \Delta t\{B_0 k_4 + B_1 k_3 + B_2 k_2 + B_3 k_1\}$

B0=2.302558088838

B1=-2.491007599848

B2=1.574340933182

B3=-0.385891422172

※ Formula for analyzing rotor system

(Ref. K.S.Brenter, "Prediction of Helicopter noise discerete frequency noise", 1986)

- Thickness noise : monopole

$$4\pi p_T' = \int_s \left[\frac{\rho_0 \dot{\vec{v}}_n}{r(1-M_r)^2} \right]_{ret} ds + \int_S \left[\frac{\rho_0 v_n \left(\dot{\vec{M}} \cdot \vec{r} + c_0 M_r - c_0 M^2 \right)}{r^2 (1-M_r)^3} \right]_{ret} ds$$

- Loading noise : dipole

$$4\pi p_L' = \frac{1}{C_0} \int_s \left[\frac{\dot{p}_l \hat{r}}{r(1-M_r)^2} \right]_{ret} dS + \int_s \left[\frac{p_{l.r} - p_l \hat{n} \cdot \vec{M}}{r^2 (1-M_r)^2} \right] dS + \frac{1}{C_0} \int_S \left[\frac{p_{l,r} \left(\dot{\vec{M}} \cdot \vec{r} + c_0 M_r - c_0 M^2 \right)}{r^2 (1-M_r)^3} \right]_{ret} ds$$

$$p'(\vec{x},t) = p_T'(\vec{x},t) + p_L'(\vec{x},t)$$

$[\ ]_{ret}$: retarded time

M_r : relative mach number

M : Mach number

r : distance between source and observer

p_l : surface pressure

v_n : velocity normal to blade surface

(determined by angular velocity & pitch angle)