< 1.4. Fundamental Aerodynamic Variables >

❖ Fundamental variables

- Throughout your working career, you will be adding to your technical vocabulary list.

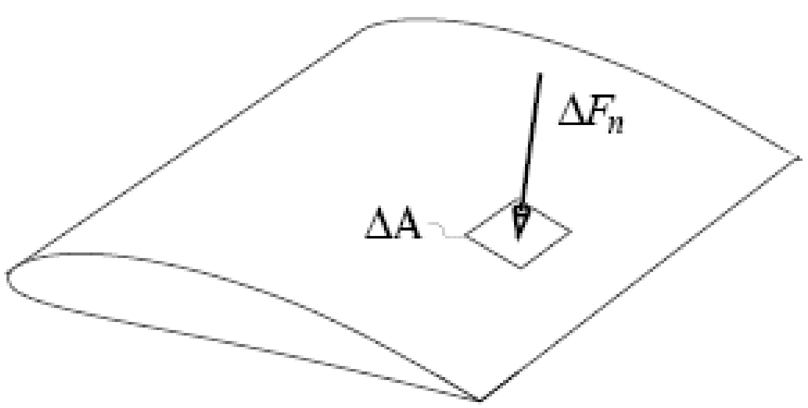
  - Pressure
  - Density
  - Temperature
  - Velocity

< 1.4. Fundamental Aerodynamic Variables >

❖ Pressure : *p*

- Definition :

  Pressure p is defined as the force/area acting normal to a surface



Normal force on area element due to pressure

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Pressure : *p*

- A solid surface doesn't actually have to be present. The pressure can be defined at any point x, y, z, in the fluid, if we assume that a infinitesimally small surface ΔA could be placed there at whim, giving a resulting normal force ΔF_n

$$p = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$

- The pressure can vary in space and possibly also time, so the pressure $p(x,y,z,t)$ in general is a time-varying scalar field.

< 1.4. Fundamental Aerodynamic Variables >

❖ Density : ρ

- Definition :

  Density ρ is defined as the mass/volume, for an infinitesimally small volume.

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta m}{\Delta v}$$

- Like the pressure, this is a point quantity, and can also change in time. So $\rho(x,y,z,t)$ is also a scalar field.

< 1.4. Fundamental Aerodynamic Variables >

❖ Temperature : *T*

- Temperature takes on an important role in high-speed aerodynamics.

- Temperature is also a point property, which can vary from point to point in the gas.

< 1.4. Fundamental Aerodynamic Variables >

❖ Velocity : *V*

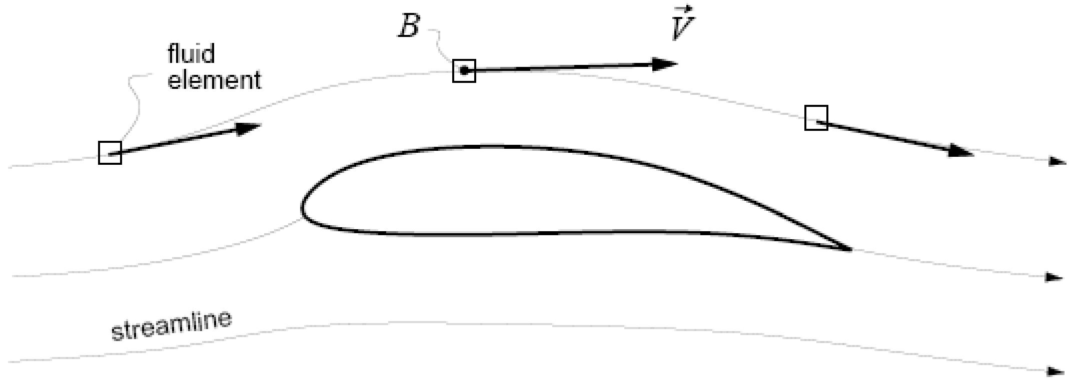
- We are interested in motion of fluids, so velocity is obviously important. Two ways to look at this:
  - Body is moving in stationary fluid – e.g. airplane in flight
  - Fluid is moving past a stationary body – e.g. airplane in wind tunnel

< 1.4. Fundamental Aerodynamic Variables >

❖ Velocity : *V*

- Consider a fluid element as it moves along. As it passes some point B, its instantaneous velocity is defined as the velocity at point B.

V at a point = velocity of fluid element as it passes that point



Velocity at point B

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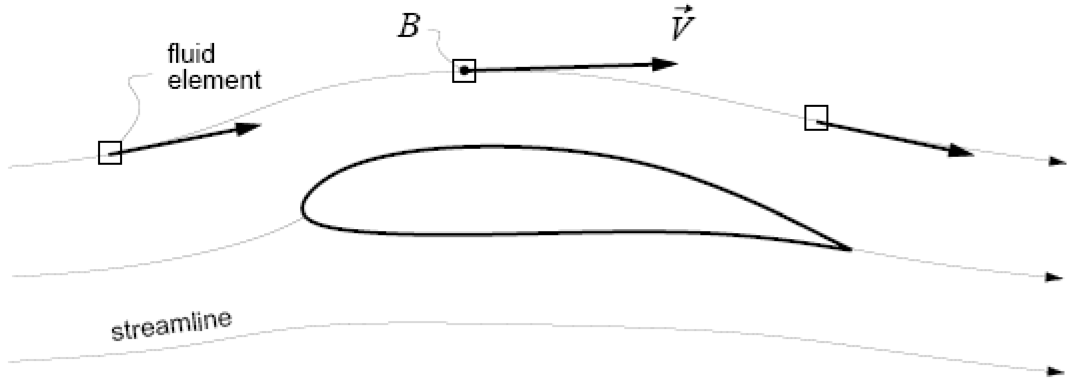
❖ Velocity : *V*

- This velocity is a vector, with three separate components, and will in general vary between different points and different times.

$$\vec{V}(x,y,z,t) = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

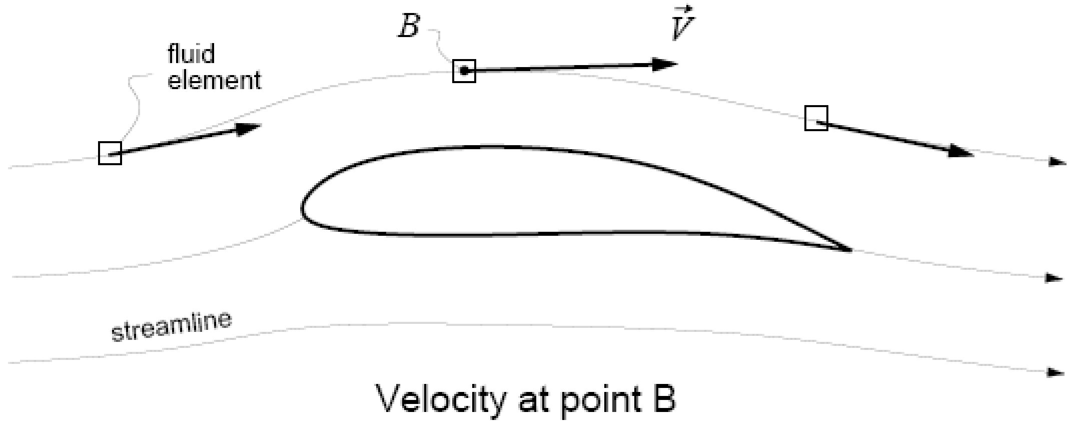


Velocity at point B

< 1.4. Fundamental Aerodynamic Variables >

❖ Velocity : *V*

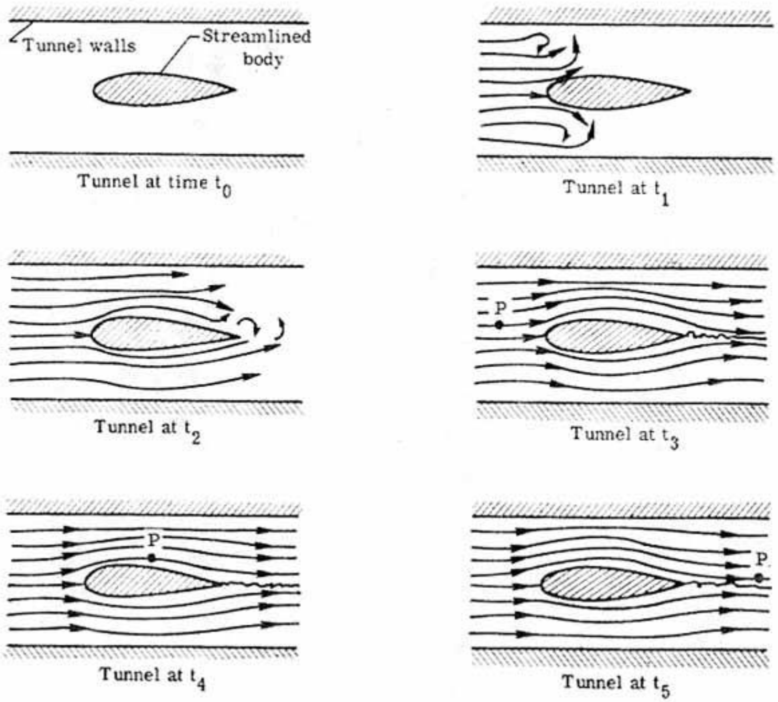
- So, ***V*** is a time-varying vector field, whose components are three separate time-varying scalar fields *u*, *v*, *w*.



< 1.4. Fundamental Aerodynamic Variables >

❖ Steady and unsteady flows

- If the flow is *steady*, then *p, ρ,* ***V*** don't change in time for any point, and hence can be given as *p(x,y,z), ρ(x,y,z),* ***V****(x,y,z)*.
- If the flow is *unsteady*, then these quantities do change in time at some or all points.



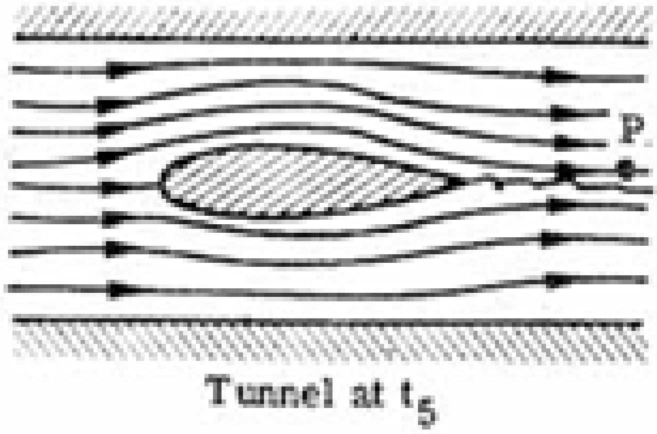
Steady flow: Particle pathline = Streamline

< 1.4. Fundamental Aerodynamic Variables >

❖ Streamline

- Definition :

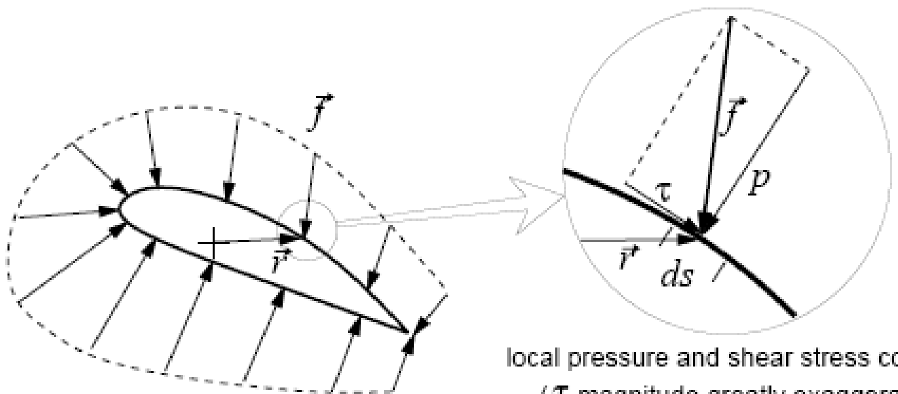
  For a steady flow, we can define a *streamline*, which is the path followed by some chosen fluid element. The figure shows three particular streamlines.



< 1.5. Aerodynamic forces and moments >

❖ Surface force distribution

- The fluid flowing about a body exerts a local force/area(or stress) ***f*** on each point of the body. Its normal and tangential components are the pressure *p* and the shear stress ***τ***.



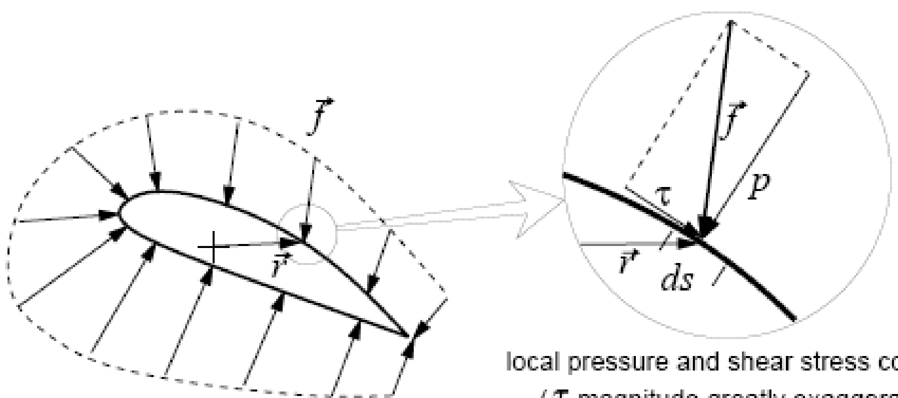
force/area distribution on airfoil

local pressure and shear stress components
(τ magnitude greatly exaggerated)

< 1.5. Aerodynamic forces and moments >

❖ Surface force distribution

- In typical aerodynamic situations, the pressure *p* is typically greater than ***τ*** by at least two orders of magnitude, and so ***f*** is very nearly perpendicular to the surface.



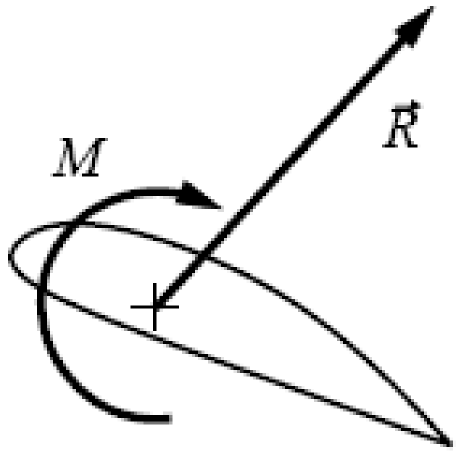
force/area distribution on airfoil

local pressure and shear stress components
(τ magnitude greatly exaggerated)

< 1.5. Aerodynamic forces and moments >

❖ Surface force distribution

- The stress distribution ***f*** integrated over the surface produces a resultant force ***R***, and also a moment ***M*** about some chosen moment-reference point. In 2-D cases, the sign convention for ***M*** is positive nose up, as shown in the figure.



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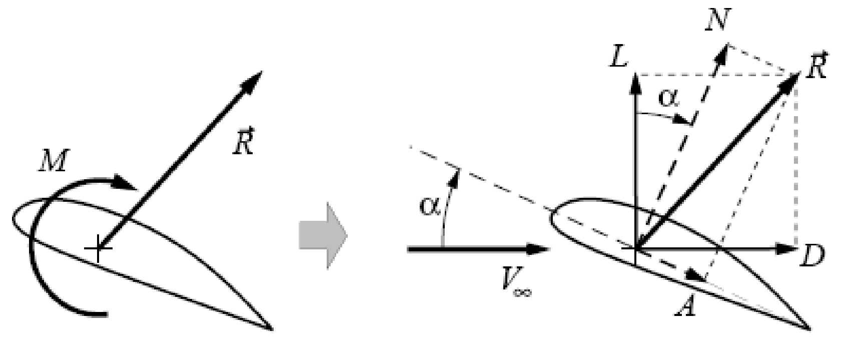
❖ Force components

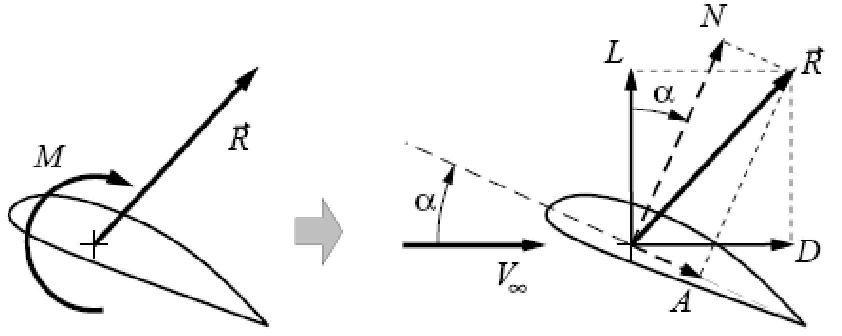


resultant force, and moment about ref. point   alternative components of resultant force

- Free-stream axes : The ***R*** components are the drag *D* and the lift *L*, parallel and perpendicular to $\boldsymbol{V}_{inf}$.
- Body axes : The ***R*** components are the axial force *A* and the normal force *N*, parallel and perpendicular to the airfoil chord line.

< 1.5. Aerodynamic forces and moments >

❖ Force components



resultant force, and moment about ref. point alternative components of resultant force

- If one set of components is computed, the other set can then be obtained by a simple axis transformation using the angle of attack α. Specifically, L and D are obtained from N and A as follows.

$$L = N\cos\alpha - A\sin\alpha$$

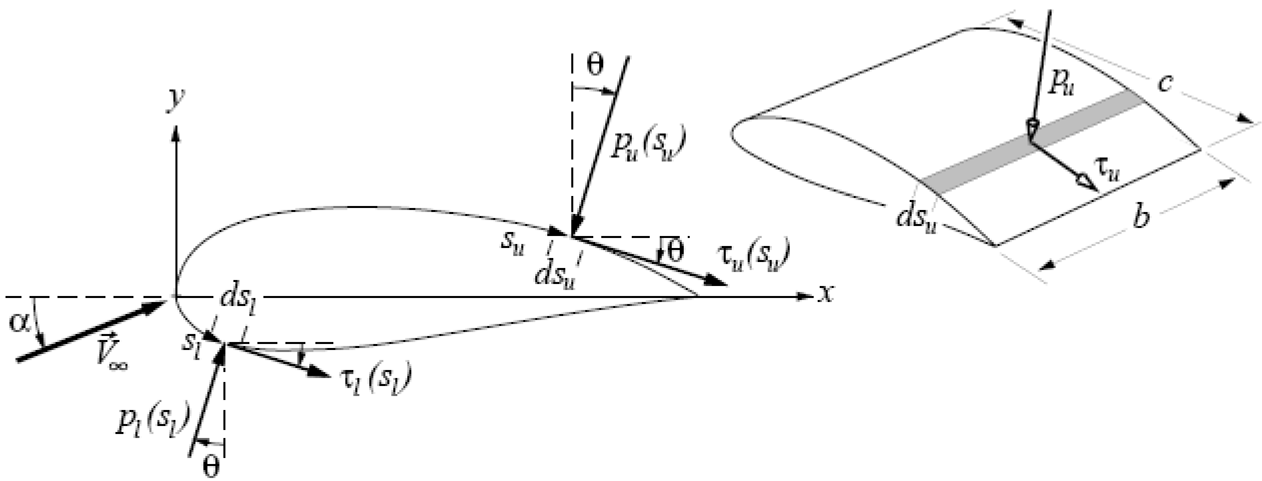
$$D = N\sin\alpha + A\cos\alpha$$

< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- A cylindrical wing section of chord *c* and span *b* has force components *A* and *N*, and moment ***M***.

$$A' \equiv A / b \qquad N' \equiv N / b \qquad M' \equiv M / b$$



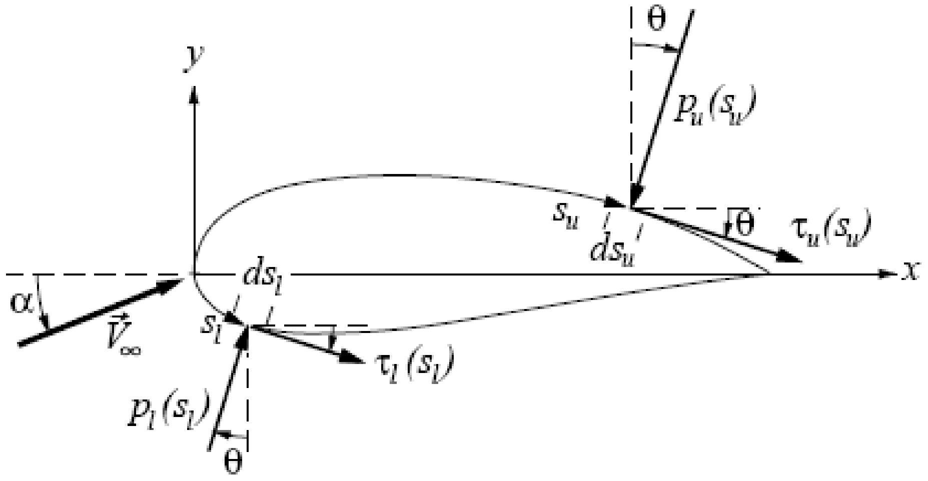
< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- On the upper surface, the unit-span force components acting on an elemental area of width ds_u are

$$dN'_u = \left(-p_u \cos\theta - \tau_u \sin\theta\right) ds_u$$

$$dA'_u = \left(-p_u \sin\theta + \tau_u \cos\theta\right) ds_u$$



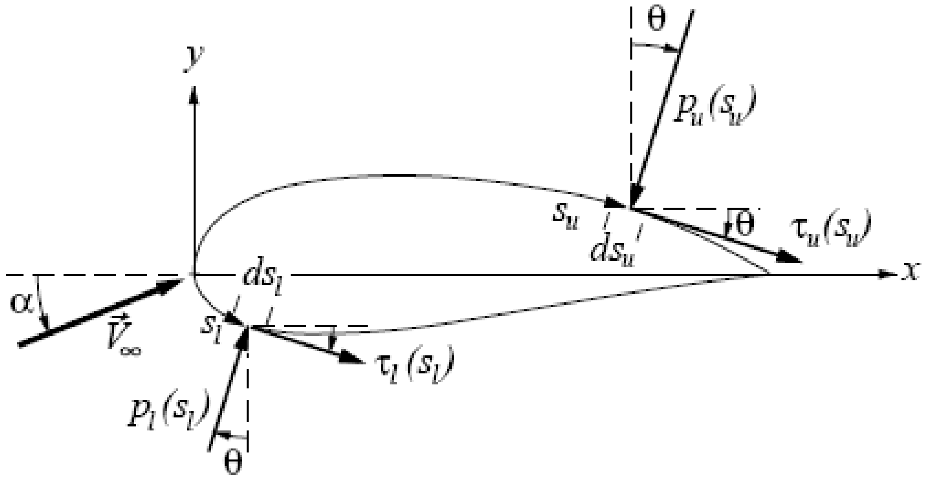
< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- On the lower surface they are

$$dN_l' = \left(p_l \cos\theta - \tau_l \sin\theta\right) ds_l$$

$$dA_l' = \left(p_l \sin\theta + \tau_l \cos\theta\right) ds_l$$



< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- Integration from the leading edge to the trailing edge points produces the total unit span forces.

$$N' = \int_{LE}^{TE} dN'_u + \int_{LE}^{TE} dN'_l$$

$$A' = \int_{LE}^{TE} dA'_u + \int_{LE}^{TE} dA'_l$$

< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- The moment about the origin (leading edge in this case) in the integral of these forces, weighted by their moment arms *x* and *y*, with appropriate sign.

$$M'_{LE} = \int_{LE}^{TE} -x\,dN'_u + \int_{LE}^{TE} -x\,dN'_l + \int_{LE}^{TE} y\,dA'_u + \int_{LE}^{TE} y\,dA'_l$$

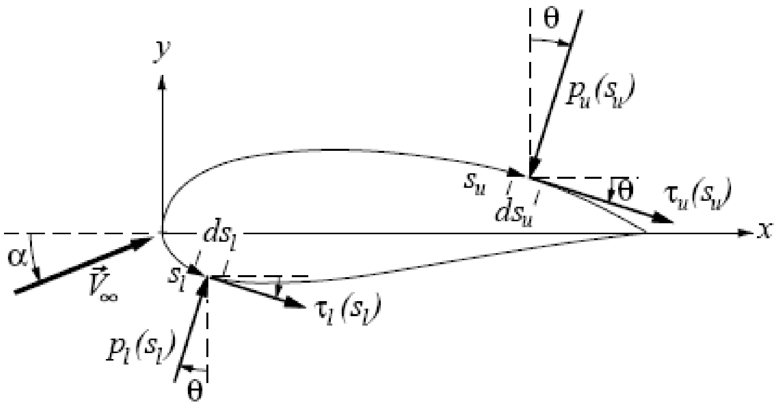
< 1.5. Aerodynamic forces and moments >

❖ Force and moment calculation

- The complete expressions are as follows :

$$M'_{LE} = \int_{LE}^{TE} \left[\left(p_u \cos\theta + \tau_u \sin\theta \right) x - \left(p_u \sin\theta - \tau_u \cos\theta \right) y \right] ds_u$$

$$+ \int_{LE}^{TE} \left[\left(-p_l \cos\theta + \tau_l \sin\theta \right) x + \left(p_l \sin\theta + \tau_l \cos\theta \right) y \right] ds_l$$



$$ds \cos\theta = dx$$

$$ds \sin\theta = -dy$$

$$= -\frac{dy}{dx} dx$$