1.4. Fundamental Aerodynamic Variables

Fundamental variables

Throughout your working career, you will be adding to your technical vocabulary list.

- Pressure
- Density
- Temperature
- Velocity
< 1.4. Fundamental Aerodynamic Variables >

- **Pressure**: $p$

  - **Definition**: Pressure $p$ is defined as the force/area acting normal to a surface.
< 1.4. Fundamental Aerodynamic Variables >

❖ Pressure: \( p \)

A solid surface doesn’t actually have to be present. The pressure can be defined at any point \( x, y, z, \) in the fluid, if we assume that a infinitesimally small surface \( \Delta A \) could be placed there at whim, giving a resulting normal force \( \Delta F_n \)

\[
p = \lim_{{\Delta A \to 0}} \frac{\Delta F_n}{\Delta A}
\]

The pressure can vary in space and possibly also time, so the pressure \( p(x,y,z,t) \) in general is a time-varying scalar field.
< 1.4. Fundamental Aerodynamic Variables >

Density : $\rho$

- Definition :

Density $\rho$ is defined as the mass/volume, for an infinitesimally small volume.

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta m}{\Delta v}$$

- Like the pressure, this is a point quantity, and can also change in time. So $\rho(x,y,z,t)$ is also a scalar field.
< 1.4. Fundamental Aerodynamic Variables >

- **Temperature :** \( T \)
  - Temperature takes on an important role in high-speed aerodynamics.
  - Temperature is also a point property, which can vary from point to point in the gas.
< 1.4. Fundamental Aerodynamic Variables >

❖ Velocity : $V$

- We are interested in motion of fluids, so velocity is obviously important. Two ways to look at this:
  - Body is moving in stationary fluid – e.g. airplane in flight
  - Fluid is moving past a stationary body – e.g. airplane in wind tunnel
1.4. Fundamental Aerodynamic Variables

**Velocity : \( V \)**

- Consider a fluid element as it moves along. As it passes some point \( B \), its instantaneous velocity is defined as the velocity at point \( B \).

\[ \text{\( V \) at a point} = \text{velocity of fluid element as it passes that point} \]
< 1.4. Fundamental Aerodynamic Variables >

**Velocity :** \( V \)

This velocity is a vector, with three separate components, and will in general vary between different points and different times.

\[
\vec{V}(x, y, z, t) = u(x, y, z, t)\hat{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k}
\]
< 1.4. Fundamental Aerodynamic Variables >

Velocity: $V$

- So, $V$ is a time-varying vector field, whose components are three separate time-varying scalar fields $u$, $v$, $w$. 
< 1.4. Fundamental Aerodynamic Variables >

- Steady and unsteady flows

- If the flow is *steady*, then $p$, $\rho$, $V$ don’t change in time for any point, and hence can be given as $p(x,y,z)$, $\rho(x,y,z)$, $V(x,y,z)$.

- If the flow is *unsteady*, then these quantities do change in time at some or all points.
Streamline

Definition:

For a steady flow, we can define a *streamline*, which is the path followed by some chosen fluid element. The figure shows three particular streamlines.
< 1.5. Aerodynamic forces and moments >

❖ **Surface force distribution**

- The fluid flowing about a body exerts a local force/area (or stress) $f$ on each point of the body. Its normal and tangential components are the pressure $p$ and the shear stress $\tau$. 
1.5. Aerodynamic forces and moments

- Surface force distribution

In typical aerodynamic situations, the pressure $p$ is typically greater than $\tau$ by at least two orders of magnitude, and so $f$ is very nearly perpendicular to the surface.
< 1.5. Aerodynamic forces and moments >

Surface force distribution

The stress distribution $f$ integrated over the surface produces a resultant force $\mathbf{R}$, and also a moment $M$ about some chosen moment-reference point. In 2-D cases, the sign convention for $M$ is positive nose up, as shown in the figure.
< 1.5. Aerodynamic forces and moments >

- Force components

- Free-stream axes: The $R$ components are the drag $D$ and the lift $L$, parallel and perpendicular to $V_{inf}$.

- Body axes: The $R$ components are the axial force $A$ and the normal force $N$, parallel and perpendicular to the airfoil chord line.
< 1.5. Aerodynamic forces and moments >

- Force components

If one set of components is computed, the other set can then be obtained by a simple axis transformation using the angle of attack $\alpha$. Specifically, $L$ and $D$ are obtained from $N$ and $A$ as follows.

\[
L = N \cos \alpha - A \sin \alpha
\]
\[
D = N \sin \alpha + A \cos \alpha
\]
< 1.5. Aerodynamic forces and moments >

- Force and moment calculation

A cylindrical wing section of chord $c$ and span $b$ has force components $A$ and $N$, and moment $M$.

$$A' \equiv A / b \quad N' \equiv N / b \quad M' \equiv M / b$$
< 1.5. Aerodynamic forces and moments >

Force and moment calculation

On the upper surface, the unit-span force components acting on an elemental area of width $ds_u$ are

\[
\begin{align*}
  dN_u' &= (- p_u \cos \theta - \tau_u \sin \theta) ds_u \\
  dA_u' &= (- p_u \sin \theta + \tau_u \cos \theta) ds_u
\end{align*}
\]
1.5. Aerodynamic forces and moments

- Force and moment calculation

On the lower surface they are:

\[ dN'_l = (p_l \cos \theta - \tau_l \sin \theta) \, ds_l \]
\[ dA'_l = (p_l \sin \theta + \tau_l \cos \theta) \, ds_l \]
< 1.5. Aerodynamic forces and moments >

- Force and moment calculation
  - Integration from the leading edge to the trailing edge points produces the total unit span forces.

\[
N' = \int_{LE}^{TE} dN'_{u} + \int_{LE}^{TE} dN'_{l}
\]
\[
A' = \int_{LE}^{TE} dA'_{u} + \int_{LE}^{TE} dA'_{l}
\]
< 1.5. Aerodynamic forces and moments >

- Force and moment calculation

The moment about the origin (leading edge in this case) in the integral of these forces, weighted by their moment arms $x$ and $y$, with appropriate sign.

$$M'_{LE} = \int_{TE}^{LE} - x dN'_u + \int_{LE}^{TE} - x dN'_l + \int_{LE}^{TE} y dA'_u + \int_{LE}^{TE} y dA'_l$$
< 1.5. Aerodynamic forces and moments >

**Force and moment calculation**

The complete expressions are as follows:

\[
M'_{LE} = \int_{LE}^{TE} \left[ \left( p_u \cos \theta + \tau_u \sin \theta \right)x - \left( p_u \sin \theta - \tau_u \cos \theta \right)y \right]ds_u \\
+ \int_{LE}^{TE} \left[ \left( - p_l \cos \theta + \tau_l \sin \theta \right)x + \left( p_l \sin \theta + \tau_l \cos \theta \right)y \right]ds_l
\]

\[ds \cos \theta = dx\]
\[ds \sin \theta = -dy\]
\[= - \frac{dy}{dx} \]